8.1

Conic Sections and Parabolas

A conic section is a cross section of a cone.

Parabola

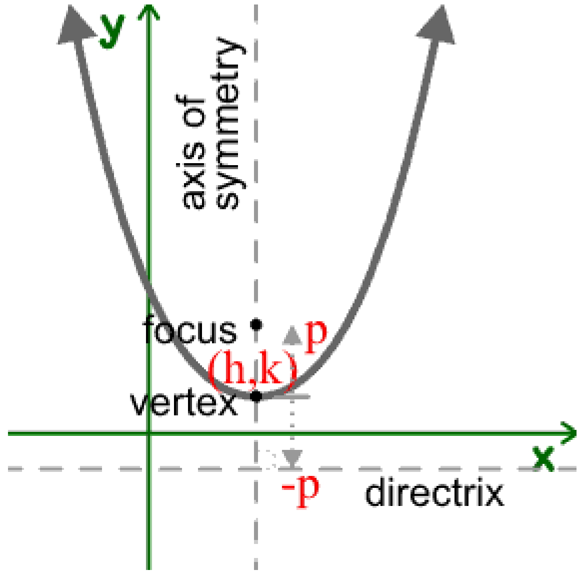
A parabola is the collection of all points P in the plane that are the same distance from a fixed point F (the **focus**) as they are from a fixed line D (the **directrix**).

The line through the focus F and perpendicular to the directrix D is called the **axis of symmetry** of the parabola. The point of intersection of the parabola with its axis of symmetry is called the **vertex** V.

Equation of a parabola; **Vertex** at (0,0), **Focus** at $(p,0)$, $p > 0$ and **Directrix** $x = -p$, $p > 0$ is:

$$y^2 = 4px.$$

In general, the points on a parabola $y^2 = 4px$ that lie above and below the focus $(p,0)$ are each at a distance $2p$ from the focus. This follows from the fact that if $x = p$, then $y^2 = 4px = 4p^2$, so $y = \pm 2p$. The line segment joining these two points is called the **focal width**; its length is $4p$.



“Discuss the equation” means find the vertex, focus, and directrix of the parabola and graph it.

Equations of a Parabola:

Vertex at (0,0), Focus on an axis; $p > 0$.

Vertex	Focus	Directrix	Equation	Description
(0, 0)	$(p, 0)$	$x = -p$	$y^2 = 4px$	Parabola, axis of symmetry is the x-axis, opens to the right
(0, 0)	$(-p, 0)$	$x = p$	$y^2 = -4px$	Parabola, axis of symmetry is the x-axis, opens to the left
(0, 0)	$(0, p)$	$y = -p$	$x^2 = 4py$	Parabola, axis of symmetry is the y-axis, opens up
(0, 0)	$(0, -p)$	$y = p$	$x^2 = -4py$	Parabola, axis of symmetry is the y-axis, opens down

Discuss: $y^2 = 8x$ $x^2 = -12y$

Find equation of parabola with focus (0, 4) and directrix y= -4 .

Parabolas with vertex at (h, k); Axis of symmetry parallel to a coordinate axis, $p > 0$.

Vertex	Focus	Directrix	Equation	Description
(h, k)	$(h + p, k)$	$x = h - p$	$(y-k)^2 = 4p(x-h)$	Parabola, axis of symmetry parallel to x-axis, opens to the right
(h, k)	$(h - p, k)$	$x = h + p$	$(y- k)^2 = -4p(x-h)$	Parabola, axis of symmetry parallel to x-axis, opens to the left

(h, k)	(h, k +*p*)	y = k - *p*	$(x-h)^2 = 4p(y-k)$	Parabola, axis of symmetry parallel to y-axis, opens up
(h, k)	(h , k - *p*)	y = k + *p*	$(x-h)^2 = -4p(y-k)$	Parabola, axis of symmetry parallel to y-axis, opens down

Find an equation of the parabola with vertex (-2,3) and focus (0,3) .

See Example 5 pg. 639

Paraboloid of revolution is a surface formed by rotating a parabola about its axis of symmetry. If a light is placed at the **focus** of the parabola, all the rays emanating from the light will reflect off the mirror in lines parallel to the axis of symmetry. This principle is used in the design of searchlights, etc.

See example 6 pg. 640